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A Large Deviation, Hamilton-Jacobi Equation Approach to a Statistical Theory for Turbulence

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6. AUTHORS

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14. ABSTRACT

2-D vortex dynamic arise naturally in helicopter turbulent models. It is important to understand its behavior.

This project is part of a long term on-going work. The goal is to develop methodology for understanding statistical behavior of complex flows by
1. computing dynamic entropy (large deviation theory) associated with stochastic systems defined by first

15. SUBJECT TERMS

Turbulent flows, large deviation, Hamilton-Jacobi equation in space of probability measures

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Report Title

ABSTRACT

2-D vortex dynamic arise naturally in helicopter turbulent models. It is important to understand its behavior.

This project is part of a long term on-going work. The goal is to develop methodology for understanding statistical behavior of complex flows by
1. computing dynamic entropy (large deviation theory) associated with stochastic systems defined by first principles
2. analyzing large particle and large time limit of the dynamic entropy to derive or mathematically characterize quasi-potentials (using Hamilton-Jacobi equations).

Novel techniques both in large deviation theory and in Hamilton-Jacobi equation theory are developed in the funded research period. Future research in this area will continue.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
2012/09/03 14	7	Deng, Feng and Liu. A singular 1-D Hamilton-Jacobi equation, with applications to large deviation of diffusions, Commun Math. Sci, (06 2011): 289. doi:
2012/09/03 14	6	Jin Feng . A Hamilton–Jacobi PDE in the space of measures and its associated compressible Euler equations, Comptes Rendus Mathematique, (09 2011): 973. doi: 10.1016/j.crma.2011.08.013
2012/09/03 13	5	Markos Katsoulakis, Jin Feng. A Comparison Principle for Hamilton–Jacobi Equations Related to Controlled Gradient Flows in Infinite Dimensions, Archive for Rational Mechanics and Analysis, (06 2008): 0. doi: 10.1007/s00205-008-0133-5
2012/09/03 13	4	Truyen Nguyen, Jin Feng. Hamilton–Jacobi equations in space of measures associated with a system of conservation laws, Journal de Mathématiques Pures et Appliquées, (04 2012): 318. doi: 10.1016/j.matpur.2011.11.004
2012/09/02 21	3	Jin Feng, David Nualart. Stochastic scalar conservation laws, Journal of Functional Analysis, (07 2008): 0. doi: 10.1016/j.jfa.2008.02.004
2011/08/27 18	1	Jin Feng, Andrzej Swiech. Optimal control for a mixed flow of Hamiltonian and gradient type in space of probability measures, Transaction of the American Mathematical Society, (01 2012): 0. doi:

TOTAL: 6

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received		Paper
2011/08/27 18	2	Truyen Nguyen, Jin Feng. Hamilton-Jacobi equations in space of measures associated with a system of conservation laws, , (01 2012): 0. doi:

TOTAL: 1

Number of Papers published in non peer-reviewed journals:

(c) Presentations

From a stochastic vortex dynamic model to Onsager-Joyce-Montgomery theory

Number of Presentations: 1.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received Paper

TOTAL:

Number of Manuscripts:

Books

Received Paper

TOTAL:

Patents Submitted

Patents Awarded

Awards

Keeler Family Professorship (Fall, 2011, University of Kansas)

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Guanlin Zhang	0.20	
FTE Equivalent:	**0.20**	
Total Number:	**1**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Jin Feng	1.00	
FTE Equivalent:	**1.00**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period:	0.00
The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:.....	0.00
The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:.....	0.00
Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):	0.00
Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:	0.00
The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense	0.00
The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:	0.00

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME
Total Number:

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Turbulent flows have strong impact to our daily life. In army operations involving helicopter, complex vortex dynamics arise naturally. It is important to understand them.

This project is focused on developing a rigorous mathematical framework to understand statistical behaviors arising from some complex flows. The original goal was just to focus on a particular type of 2-D vortex flows known as the Onsager-Joyce-Montgomery theory. Significant progress on this problem has been made with a complete solution expected to be finished soon. During the research process, a bigger structure has emerged. New methodologies are introduced which has potential to work for some multi-D irrotational compressible flows related to Euler equation. The details will be explored in the future.

Specifically, during the funded period,

1. a rigorous approach to verify the Onsager-Joyce-Montgomery theory for 2-D vortex dynamic is proposed. A controlled PDE (variational problem defined on curves in space of probability measures) is rigorously analyzed (joint article with Andrzej Swiech, accepted by Transaction of American Mathematical Society). Further works involves rigorously establishing a large deviation principle to give dynamic version of entropy (i.e. energy functional in the variational problem) for underlying stochastic model, and for analyzing large time behavior of the entropy. These on-going works are being pursued now and will appear in the near future.

2.well-posedness of stochastic scalar conservation laws with multiplicative noise was given. This is the first results for this kind of equation. The well-posedness of deterministic scalar conservation law equation was first given by Kruzkov in early 1970s, through the introduction of entropy solution. In an article (joint with David Nualart, Journal of Functional Analysis 2008), the right generalization to stochastic case was discovered to enable the uniqueness proof to stochastic setting.

3. the P.I. and collaborators introduced a new class of equations (Hamilton-Jacobi equation in space of probability measures) to study large deviation and controlled PDEs motivated from mechanics. The first result of its kind for well-posedness of this class of equation is proved using theory of optimal mass transportation techniques. Subsequent works to systematically develop this into a bigger picture are currently under its way (joint works with Thomas G. Kurtz, Markos Katsoulakis and Andrzej Swiech). It is found that such equations also arise from the recent Lasry-Lions mean-field games.

4. A particular type of compressible irrotational flows is derived as action minimizer for a variational problem related to probabilistic large deviation of interacting particle systems. In particular, a Hamilton-Jacobi equation in space of probability measure formulation of the Hamiltonian-Lagrangian formalism is given and the Hamilton-Jacobi PDE is shown to be well-posed. (joint work with T Nguyen, Journal de Mathematique Pures et Appliquees). Future works focusing on large time behavior for such equations is currently under its way.

Technology Transfer